University of Global Village

Dept. of EEE,

Dr. Golam Salem

Circuits for D.C Measurements ,Precise Resistance Measurement

Objectives

After completing this chapter, you should be able to:

- Describe the construction and operation of bridge circuit.
- Understand how to measure the different values of resistances using different methods.
- Explain the measurement of resistance with Ammeter-voltmeter method
- Describe the Potentiometer Method.
- Explain in detail the working of Wheatstone, Kelvin and Megaohm Bridges
- Understand the Carey-Foster slide-wire bridge method
- Explain the working of Megger

4.1 Introduction

A resistor is an electrical component, which has been manufactured with specific amount of resistance. In electronic circuit, the resistor plays one of the most important part. The resistors are used mainly for two purposes, namely controlling the flow of electric current and providing desired amounts of voltage in electric or electronic circuits.

There are different methods available for measuring the resistance. The precision measurements of component values have been made by using various forms of bridges and instruments. There are different types of resistor according to their resistance value. So we use different instrument to measure their resistance value.

4.2 Bridge Circuit

Fig. 4.1 shows the circuit diagram of a simple bridge circuit. A **bridge circuit** is a type of electrical circuit in which two circuit branches (usually in parallel with each other) are connected (bridged) by a third branch between the first two branches at some intermediate point along them. Bridge circuits are used in many applications, both linear and non-linear, including instrumentation, filtering and power conversion.

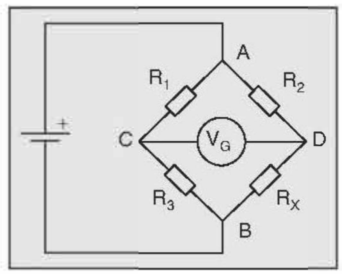

Fig. 4.1.

The bridge circuit constructed from single or combination of passive circuit elements (resistors, inductors, capacitors). Let us consider that the bridge circuit construct from the resistors element, one of which has an unknown value (R_x), and known resistance R_1, R_2 and R_3, where R_3 is the variable resistance. The two opposite corners of the square are connected to a source of electric current, such as battery, while the galvanometer is connected across the other two opposite corners. The current through the galvanometer depends on the potential difference between the two point C and D. To find the value of unknown resistor we have to first balance the bridge circuit. The bridge is balanced when there is no current flow through the galvanometer. The potential difference across the galvanometer is zero.

The variable resistor is adjusted until the galvanometer reads zero. Then the ratio between the variable resistor and its neighbor is equal to the ratio between the unknown resistor and its neighbour, and this enables the value of the unknown resistor to be calculated.

$$\frac{R_1}{R_2} = \frac{R_2}{R_X}$$

The simplest form of bridge is for the purpose of measuring resistance and is called Wheatstone bridge.

4.3 Advantages of Bridge Circuits

Although there are several advantages of bridge circuits, yet the following one s are important from the subject point of view:

1. Accuracy in high measurements.
2. Accuracy is independent of null detector characteristics.
3. It can be used in control circuits.

4.4 Types of Bridge Circuit

The bridge circuits are of two types DC Bridge and AC Bridge. In DC bridge circuit, a DC source battery and a galvanometer are used. While in the AC bridge circuit, an AC source and a detector sensitive to AC voltage are used. Further the bridges are classified in many types. The classification of bridge depend on the resistance value, element of unknown branches, Q value etc. The bridge circuit may be broadly classified into the following categories:

1. D.C. Bridge Circuits

Different types of DC bridge circuits are given below :

(*a*) Wheatstone Bridge Circuit
(*b*) Kelvin Bridge Circuit
(*c*) Double Kelvin Bridge Circuit

2. A.C Bridge Circuits

Difference types of AC bridge circuits are given below :

(*a*) Maxwell Bridge Circuit
(*b*) Hay's Bridge Circuit
(*c*) Anderson's Bridge Circuit
(*d*) Owen's Bridge Circuit
(*e*) De Sauty's Bridge Circuit
(*f*) Schering Bridge Circuit
(*g*) Wien Bridge Circuit
(*h*) Resonance Bridge Circuit

The DC Bridge is covered in this chapter while the AC Bridge, in the next chapter.

4.5 Classification of Resistances

Based upon the value, the resistances can be classified into the following three categories:

1. **Low resistance.** The resistance with values less than or equal to 1 Ω are called low resistance. Armatures winding of machines, ammeter shunts, cables, contacts etc. all have a low resistance value.
2. **Medium resistance.** The resistance with values ranging from 1 Ω to 100 kΩ are called medium resistance. The resistors employed in electronic circuits usually are of medium resistance type.
3. **High resistance.** The resistance with values above 100 kΩ are high resistances. Some of the electrical and electronic circuits do employ resistors with high resistance values.

4.6 Measurement of Resistance

There are several methods used for the measurement of resistance. Fig. 4.2 shows the methods that are used for the measurement the low-, medium- and high-resistance values.

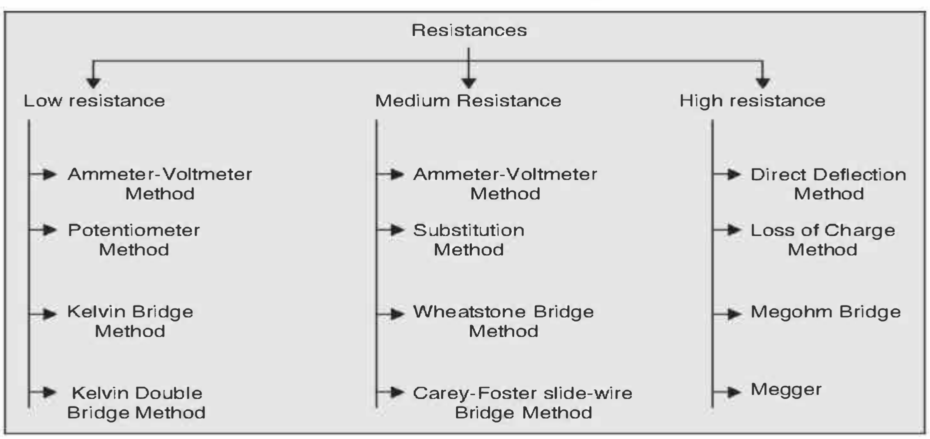

Fig. 4.2.

4.1 Measurements of Low Resistance

There are several methods for the measurement of low resistance value. But the following ones are important from the subject point of view :

1. Ammeter-Voltmeter Method
2. Potentiometer Method
3. Kelvin Bridge Method
4. Kelvin Double Bridge Method

We shall now discuss all the above mentioned methods one by one in the following pages.

4.2 Ammeter-Voltmeter Method

This method is used for measuring low resistance value when accuracy of the order of 1 % is sufficient. The ammeter-voltmeter method employs the simple ohm's law to determine the value of an unknown resistance.

Fig. 4.3 shows the circuit arrangement of ammeter-voltmeter method. The current through the unknown resistor (R_X) and potential drop across it are measured simultaneously. The readings are obtained by ammeter and voltmeter respectively. The required range of instrument to be used and the voltage of the supply required will depend on the size and rating of the resistance under test. A high-value resistor will require high-voltage source, a high-range voltmeter and a low-range ammeter whereas, a low-value resistor will require in most cases, a low–voltage, high-current source, a low-range voltmeter and a high-range ammeter. The exact requirement will, of course, depend also on the rating of the resistor, as well as the instruments available. There are two ways to connect the voltmeter as discussed below.

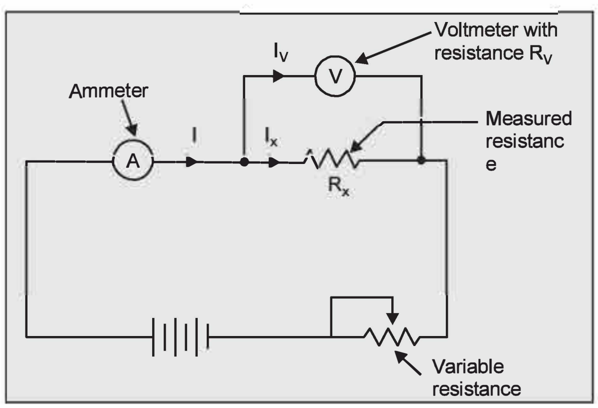

Fig. 4.3.

1. *Voltmeter is connected directly across the resistor only*

When the voltmeter is connected directly across the resistor, the ammeter measures the current flowing through the unknown resistance R_X and the voltmeter.

Current through ammeter

= Current through unknown resistance (X) + Current through voltmeter

$$I = I_X + I_V$$

$$I_X = I - I_V$$

the value of unknown resistance,

$$R_X = \frac{V}{I_X} = \frac{V}{I - I_V} = \frac{V}{I - V/R_V}$$

$$= \frac{V}{I\left(1-\frac{V}{IR_V}\right)} \quad \text{----(i)}$$

where V = voltmeter reading

R_V = resistance of the voltmeter

I = current indicated by ammeter

The value of unknown resistance is determined by equation (*i*).

Substituting the value, $V/I = R_m$ in the equation (*i*) we get,

$$R_X = R_m \left(\frac{1}{1-\frac{R_m}{R_V}} \right)$$

From the above equation we see that the true value of unknown resistance is equal to measured value of unknown resistance provided that voltmeter is of infinite resistance. However if the voltmeter is of very large resistance as compared to the resistance under measurement then,

i.e. $R_V >> R_m$

$$R_X = 1 + \frac{R_m}{R_V}$$

Thus the measured value of unknown resistance, R_m is lesser than its true value.

2. *Voltmeter is connected directly across the ammeter and resistor:*

Figure 4.4 shows the voltmeter connected directly across the ammeter and unknown resistance, R_X, the voltmeter measures the voltage drop across the ammeter and unknown resistance. The ammeter is connected so that it indicates only the current flowing through the unknown resistance.

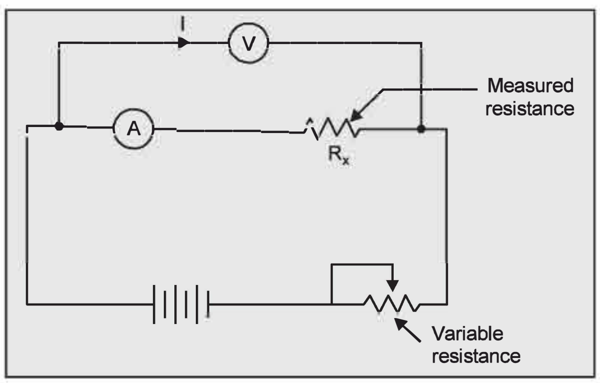

Fig. 4.4.

$$V = IR_A + IR_X = I(R_A + R_X)$$

$$R_X = \frac{V}{I} - R_A \quad \ldots(ii)$$

where R_A is the resistance of the ammeter. The value of unknown resistance is determined by the equation (*ii*).The ammeter-voltmeter method of measuring resistance is capable of fair accuracy, depending on care in taking the reading and on the accuracy and range of the instruments used for measurement of voltage and current. This method is useful in some laboratory work in which high accuracy is not required.

4.3 Advantages and Disadvantages of Ammeter-Voltmeter Method

Advantages

Some of the main advantages of ammeter-voltmeter method are given below :

1. It does not require skilled operation.
2. Accuracy of the order ± 1 % can be achieved.

Disadvantages

Some of the main disadvantages of ammeter-voltmeter are given below :

1. A correction factor needs to apply on the measured value to obtain the true value of the resistance.
2. The low values of resistances invariably have a high percentage of error.

4.4 Potentiometer Method

In potentiometer method the unknown resistance is compared with a standard resistance of the same order of magnitude. Fig. 4.5 shows the circuit diagram of potentiometer method. As seen from the diagram, the unknown resistance R_X, ammeter A, a rheostat R (to limit the current) and a standard resistance are connected in series with low voltage high current supply source. The value of standard resistance should be known.

The current through the circuit is adjusted by a rheostat so that a potential difference across the resistor is about 1V. The voltage drop across the potentiometer and the standard resistor is measured by a potentiometer. The ratio of the two potentiometer reading gives the ratio of R_X to S.

$$\frac{R_X}{S} = \frac{\text{Potentiometer reading across } R_X}{\text{Potentiometer reading across } S} = \frac{V_X}{V_S}$$

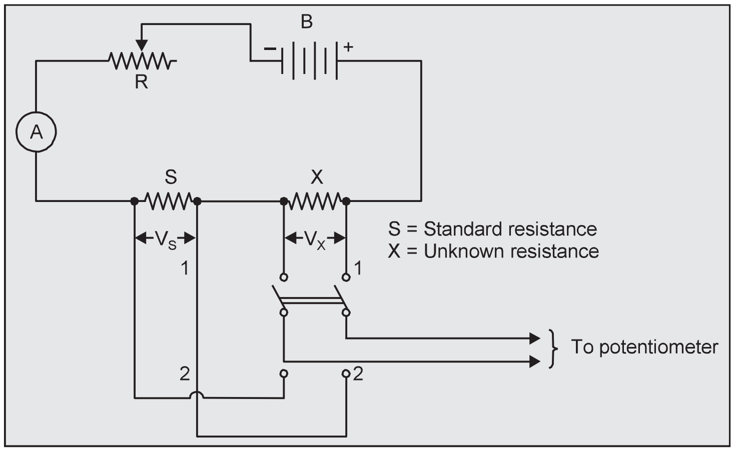

Fig. 4.5.

The accuracy of this method depends upon there being no change in current between the two readings. The source to supply current through the circuit should be extremely stable.

4.5 Advantages and Disadvantages of Potentiometer Method

Advantages of Potentiometer Method

Though there are numerous advantages of potentiometer method, yet some of the important are given below:

1. Inexpensive
2. Simple to handle
3. Useful for the measurement of large amplitudes of displacement
4. Electrical efficiency is very high.

Disadvantages of Potentiometer Method

Though the potentiometer method has a number of disadvantages, yet some of them are given below:

1. Force is required to move the sliding contacts.
2. Sliding contacts can wear out, become misaligned and generate noise.

4.6 Kelvin Bridge

Fig. 4.6 shows the circuit diagram of a Kelvin Bridge. This circuit provides great accuracy in the measurement of low value resistance generally below 1 Ω. It is used for measuring resistance values ranging from microohms to 1 ohm.

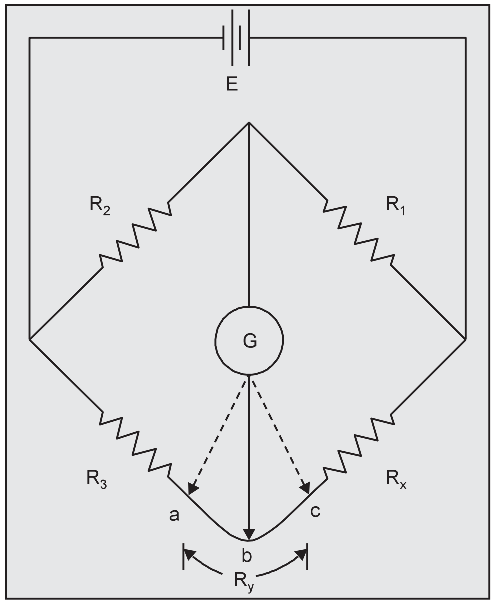

Fig. 4.6. Kelvin Bridge

The resistance R_y represents the resistance of the conducting lead from R_3 to R_x. The resistance R_x is the unknown resistance to be measured. The galvanometer can be connected either to point '*c*' or to point '*a*'. When it is connected to point '*a*', the resistance R_y of the connecting lead is added to the unknown resistance R_x. The measurement value of the resistance is too high than the actual value.

When the galvanometer is connected to the point '*c*', the resistance R_y of the connecting lead is added to the known resistance R_3. The actual value of R_3 is higher than the normal value by the resistance R_y and the resulting measurement of R_x is lower than the actual value.

If the galvanometer is connected to point '*b*', in between points '*c*' and '*a*', in such a way that the ratio of the resistance from '*c*' to '*b*' and that from '*a*' to '*b*' equals the ratio of resistance R_1 and R_2 then,

$$\frac{R_{cb}}{R_{ab}} = \frac{R_1}{R_2}$$

Balance equation for the bridge is given by relation,

$$\frac{R_x + R_{cb}}{R_3 + R_{ab}} = \frac{R_1}{R_2}$$

$$(R_x + R_{cb}) = \frac{R_1}{R_2}(R_3 + R_{ab}) \qquad \ldots(i)$$

We know that

$$R_{ac} + R_{bc} = R_y$$

and $$\frac{R_{bc}}{R_{ac}} = \frac{R_1}{R_2} \quad \ldots(ii)$$

Adding 1 on the both side of equation (*ii*) we get

$$\frac{R_{bc}}{R_{ac}} + 1 = \frac{R_1}{R_2} + 1$$

$$\frac{R_{bc} + R_{ac}}{R_{ac}} = \frac{R_1 + R_2}{R_2}$$

$$\frac{R_y}{R_{ac}} = \frac{R_1 + R_2}{R_2} \quad \ldots(R_{ac} + R_{bc} = R_y)$$

$$R_{ac} = \frac{R_2 R_y}{R_1 + R_2} \quad \ldots(iii)$$

$$R_{bc} = R_y - R_{ac}$$

$$= R_Y - \frac{R_2 R_Y}{R_1 + R_2}$$

$$= \frac{R_1 R_y}{R_1 + R_2} \quad \ldots(iv)$$

Substituting the equation (*iii*) and (*iv*) in equation (*i*),

$$R_x + \frac{R_1 R_y}{R_1 + R_2} = \frac{R_1}{R_2}\left(R_3 + \frac{R_2 R_y}{R_1 + R_2}\right)$$

$$R_x = \frac{R_1 R_3}{R_2}$$

This is the standard equation of the bridge balance. The equation does not depend on the resistance of connecting lead from R_3 to R_x. The effect of lead and contact resistances is completely eliminated by connecting the galvanometer to the intermediate position '*b*'.

4.7 Double Kelvin Bridge

Fig. 4.7 shows the circuit diagram of Kelvin double bridge. This bridge contains another set of ratio arms hence called double bridge. The second set of arms labeled '*l*' and '*m*'. The galvanometer is connected to point '*f*'. The ratio of the resistances of arms '*l*' and '*m*' is same as the ratio of R_1 and R_2.

The galvanometer indicates "zero" when the potential at '*a*' equals the potential at '*f*', i.e.,

$$E_{ab} = E_{bcf}$$

According to the *Voltage Divider Rule* the voltage across the '*ab*',

$$E_{ab} = \frac{R_2}{R_1 + R_2} \times E \quad \ldots(i)$$

The value of *E* is given by,

$$E = I\,[R_3 + R_x + (l + m) \,||\, R_y]$$

$$E = I\left[R_3 + R_x + \frac{(l + m)\,R_y}{(l + m) + R_y}\right]$$

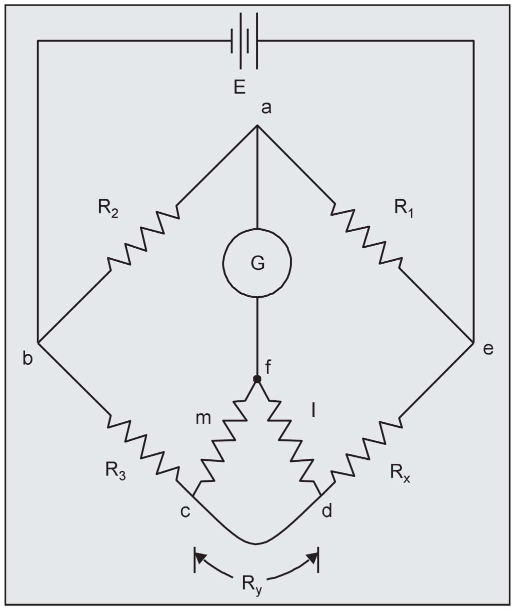

Fig. 4.7. Double Kelvin Bridge

Substituting the value of E in equation (i) we get,

$$E_{ab} = \frac{R_2}{R_1 + R_2} \times I \left[R_3 + R_x + \frac{(l+m)\,R_y}{(l+m)+R_y} \right]$$

Similarly,

$$E_{bcf} = I \left[R_3 + \frac{m}{l+m} \left\{ \frac{(l+m)\,R_y}{(l+m)+R_y} \right\} \right]$$

but,

$$E_{ab} = E_{bcf} = \frac{R_2}{R_1 + R_2} \times I \left[R_3 + R_x + \frac{(l+m)\,R_y}{(l+m)+R_y} \right]$$

Rearranging the above equation we get,

$$R_x = \frac{R_1\,R_3}{R_2} + \frac{m R_1\,R_y}{R_2\,(a+b+R_y)} - \frac{l R_y}{l+m+R_y}$$

$$R_x = \frac{R_1\,R_3}{R_2} + \frac{mR_y}{l+m+R_y}\left(\frac{R_1}{R_2} - \frac{l}{m}\right) \qquad \ldots(ii)$$

The ratio of the resistances of arms 'l' and 'm' is same as the ratio of R_1 and R_2, i.e.,

$$\frac{R_1}{R_2} = \frac{l}{m} \qquad \ldots(iv)$$

From equation (iv) and (iii) we get the value of R_x,

$$R_x = \frac{R_1\,R_3}{R_2}$$

This is the equation for Kelvin Bridge. It indicates that the resistance of the connecting lead R_Y, has no effect on the measurement, provided that the ratios of the resistances of the two sets of ratio arms are equal. Fig. 4.8 shows the Kelvin double bridge. This bridge is mostly used for industrial and laboratory purpose.

Fig. 4.8. Kelvin double bridge used in industry

Example 4.3. *In a Kelvin double bridge, there is error due to mismatch between the ratio of outer and inner arm resistances. The following data relates to this bridge,*

Standard resistance = 100.03 μΩ

Inner arms = 100.31 Ω and 200 Ω

Outer arms = 100.24 Ω and 200 Ω

The resistance of connecting leads from standard to unknown resistance is 680 μΩ. Determine the value of unknown resistance.

Solution. Given: $R_3 = 100.03\ \mu\Omega = 100.03 \times 10^{-6}\ \Omega$; $l = 100.31\ \Omega$; $m = 200\ \Omega$; $R_1 = 100.24\ \Omega$; $R_2 = 200\ \Omega$ and $R_y = 680\ \mu\Omega = 680 \times 10^{-6}\ \Omega$.

We know that the value of unknown resistance

$$R_x = \frac{R_1 R_3}{R_2} + \frac{m R_y}{l + m + R_y}\left(\frac{R_1}{R_2} - \frac{l}{m}\right)$$

$$= \frac{100.24 \times (100.03 \times 10^{-6})}{200} + \frac{200 \times (680 \times 10^{-6})}{100.31 + 200 + (680 \times 10^{-6})}\left(\frac{100.24}{200} - \frac{100.31}{200}\right)$$

$= (50.135 \times 10^{-6}) + (4.528 \times 10^{-4}) \times (-3.5 \times 10^{-4})$

$= 49.97 \times 10^{-6}\ \Omega$

4.8 Measurement of Medium Resistance Values

There are several methods for the measurement of medium resistance values. Some of the important methods different types of method are as given below.

Ammeter-voltmeter Method, Substitution Method, Carey-Foster slide-wire Bridge Method.

Wheatstone Bridge

The Wheatstone bridge circuit is used to compare an unknown resistance with a known resistance. The bridge is commonly used in control circuits. Wheatstone bridge is the most accurate method available for measuring resistance and is popular for laboratory use.

Fig. 4.14 shows the Wheatstone bridge circuit. The bridge has four resistive arms, together with a source of e.m.f and a null detector. The source of e.m.f and a switch is connected to '*A*' and '*B*', while a current indicating meter, "galvanometer" is connected between '*C*' and '*D*'. When there is no current through the galvanometer, the pointer shows zero. The current in one direction cause the pointer to deflect on one side and current in the opposite direction cause the pointer to deflect to the other side.

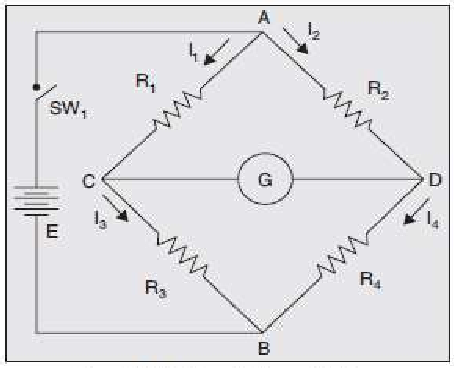

Fig. 4.14 Wheatstone Bridge.

The bridge is said to be balanced when the potential difference across the galvanometer is '0V' so that there is no current through the galvanometer. This condition occurs when the voltage from point '*C*' to point '*A*' is equal to the voltage from point '*D*' to point '*A*'. By referring the other battery terminal, the bridge is balanced when the voltage from point '*C*' to point '*B*' equals the voltage from point '*D*' to point '*B*'. Thus when the bridge is balanced,

$$I_1 R_1 = I_2 R_2 \qquad \ldots(i)$$

Applying Kirchhoff's Voltage Law in loop *ABC* when the galvanometer current is zero, we get,

$$I_1 R_1 + I_3 R_3 - E = 0$$

But since current $I_1 = I_3$, we get,

$$I_1 = I_3 = \frac{E}{R_1 + R_3} \qquad \ldots(ii)$$

Similarly, in loop *ADB* we get,

$$I_2 = I_4 = \frac{E}{R_2 + R_4} \qquad \ldots(iii)$$

Using equation (*i*) (*ii*) and (*iii*), we get

$$\frac{R_1}{R_1 + R_3} = \frac{R_2}{R_2 + R_4}$$

or

$$R_1 R_4 = R_2 R_3 \qquad \ldots(iv)$$

In balanced condition if three of the resistances have known values, then the value of fourth resistance is calculated from the equation (*iv*). If R_4 is unknown resistor R_x, then the value of R_x,

$$R_x = R_3 \frac{R_2}{R_1}$$

Resistor R_3 is called the standard arm of the bridge, and resistors R_2 and R_1 are called the ratio

Example 4.4. *In a Wheatstone bridge resistance R_1 = 20 kΩ, R_2 = 30 kΩ and R_3 = 80 kΩ. Determine the unknown resistance R_X.*

Solution. Given: R_1 = 20 kΩ; R_2 = 30 kΩ and R_3 = 80 kΩ.

We know that the value of unknown resistance in Wheatstone bridge,

$$R_x = R_3 \frac{R_2}{R_1}$$

$$= \frac{30 \times 80}{20} = 120 \text{ k}\Omega \textbf{ Ans.}$$

4.18 Unbalanced Wheatstone Bridge

To determine whether or not the galvanometer has the required sensitivity to detect an unbalance condition, it is necessary to calculate the galvanometer current. To determine the amount of deflection that would result for a particular degree of unbalance, we use Thevenin's theorem,

Fig. 4.16 shows the unbalanced Wheatstone bridge, since we are interested in the circuit through the galvanometer, the Thevenin equivalent circuit is determined by looking into galvanometer terminals '*c*' and '*d*'.

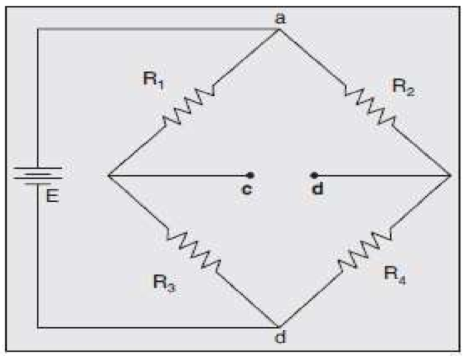

Fig. 4.16. Unbalanced Wheatstone Bridge.

The Thevenin or open-circuit voltage is given by,

$$E_{th} = E_{ac} - E_{ad} = I_1 R_1 - I_2 R_2$$

where

$$I_1 = \frac{E}{R_1 + R_3}$$

and

$$I_2 = \frac{E}{R_2 + R_4}$$

$$E_{th} = E\left(\frac{R_1}{R_1 + R_3} - \frac{R_2}{R_2 + R_4}\right)$$

The resistance of the Thevenin equivalent circuit is found by looking back into terminals *c* and *d* and replacing the battery by its internal resistance. Thus Thevenin resistance looking into terminals *c* and *d*,

$$R_{th} = \frac{R_1 R_3}{R_1 + R_3} + \frac{R_2 R_4}{R_2 + R_4}$$

The galvanometer current is given by,

$$I_g = \frac{E_{th}}{R_{th} + R_g}$$

where I_g is the current through galvanometer and R_g is its resistance.

NUMERICAL PROBLEMS

1. The ammeter-voltmeter method is used to measure a resistance. With the voltmeter connected across the resistance the reading on the ammeter and voltmeter are 0.4 A and 3.2 V respectively. The resistance of the voltmeter is 500 Ω. Calculate (*i*) true value of resistance. (*ii*) Percentage error in the value of resistance, if the voltmeter current is ignored.

(**Ans.** 8.13 Ω; 1.6 %)

2. In a specific Kelvin bridge the ration of the arms are 1:100. The standard resistance is 20 Ω. Find out the unknown resistance.

(**Ans.** 0.2 Ω)

3. The insulation resistance of 2 metre cable was measured by loss of charge method. The voltage across the standard capacitance of 0.003 pF drops from 222 V to 155 V in 1 minute. Calculate the insulation resistance of the cable.

(**Ans.** 55,670 MΩ)

4. In a measurement of resistance by potentiometer, the voltage drops across a resistor under test and across 0.02 Ω standard resistor were found to be 0.735 V and 9.8 V respectively. Determine the value of resistor under test.

(**Ans.** 0.015 Ω)

5. Determine the insulation resistance of a short length of cable in which voltage falls from 125 to 100 volts in 25 seconds. The capacity of the condenser is 600×10^{12} F.

(**Ans.** 1,86,726 MΩ)

6. The Wheatstone bridge consists of the following parameters: R_1 = 10 kΩ R_2 = 15 kΩ and R_3 = 40 kΩ. Fine the unknown resistance R_X.

(**Ans.** 60 Ω)